THERMAL RELAXATION AND NONLINEAR MELTING AND EVAPORATION IN INTENSE ENERGY FLUXES

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An analytic study has been made on thermal relaxation in a nonlinear medium showing phase transitions consequent on high-power surface energy sources.

There are many papers on simulating heat transfer in transitions produced by concentrated energy fluxes; see [1-5] for the state of the art and an extensive bibliography. A major aspect of high-intensity nonstationary thermal processes is that heat propagates at a finite rate, which influences the temperature pattern [6, 7], as for example in metal phase transitions at sufficiently high incident fluxes [4, 8]. Various analytic methods have been applied [8, 13].

I have examined new classes of analytic solution within the framework of a Stefan heat-transfer treatment for melting and evaporation with allowance for heat-flux relaxation.

1. Initial Equations. We use a dimensionless form for the equations governing one-dimensional nonstationary heat transfer with relaxation [6, 7]:

$$\Omega c T_t + q_x = 0, \quad \Omega\lambda T_x + q + \gamma q_t = 0,$$
$$\Omega = \lambda_b T_b / q_b x_b, \quad x_b/t_b = (\lambda_b/\gamma_b c_b)^{1/2}. \tag{1}$$

Here $\overline{T} = TT_b$, $\overline{q} = qq_b$, etc. The time scale is taken as the heat-flux relaxation period $t_b = \gamma \equiv$ const, while the dimensionless quantity $\gamma = 1$ is retained in the formulas for clarity.

We introduce the new argument $\tau = \exp(-t/\gamma)$ and represent (1) as

$$\Omega u_\tau = v_x, \quad \tau^2 v_\tau = \Omega a\gamma u_x,$$
$$a = \lambda/c, \quad u'(T) = c(T), \quad q = \tau v/\gamma. \tag{2}$$

2. Nonstationary Melting. We take $\Omega = 1$, $u = \psi_x$, $v = \psi_\tau$ and replace (2) by a second-oder differential equation for $\psi = \psi(x, \tau)$:

$$\psi_{\tau\tau} = (a\gamma/\tau^2)\,\psi_{xx}, \tag{3}$$

where the thermal diffusivity a is a function of ψ_x. We proceed by analogy with [12] and transfer from (3) to the Monje–Ampère equation by means of the Legendre transformation $F(u, \tau) = xu - \psi(x, \tau)$, $x = F_u$, $v = -F_\tau$, whereupon from [14, 15] we can show that in the case

$$a = a_1/(u + k_1)^2, \quad u + k_1 \neq 0, \quad a_1,\ k_1 - \text{const} \tag{4}$$

there is a parametric exact solution for (1):

$$x(u, \tau) = \zeta'(\omega) + [f_1/(u + k_1)], \quad \omega = (u + k_1)/\tau, \tag{5}$$

$$\gamma q(u, \tau) = f_1 - \tau\zeta(\omega) + (u + k_1)\,\zeta'(\omega), \quad a_1\gamma = f_1^2, \tag{6}$$

in which $\zeta(\omega)$ is an arbitrary function. There is a marked difference between (5) and (6) on the one hand and the approximate solutions (local in t) [12] in that the time dependence is of relaxation type.

Power-law ($k_1 = 0$) and exponential forms ($k_1 n_2 = c_1$) can be given for the thermophysical parameters satisfying (4):

$$\lambda = \lambda_1 T^{n_1}, \quad c = c_1 T^{n_2}, \quad n_1 + n_2 = -2, \quad \lambda_1 c_1 \gamma = f_1^2 (1 + n_2)^2; \tag{7}$$

$$\lambda = \lambda_1 \exp(n_1 T), \quad c = c_1 \exp(n_2 T), \quad n_1 + n_2 = 0, \quad \lambda_1 c_1 \gamma = f_1^2 n_1^2. \tag{8}$$

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For example, for $n_1 < 0$, $n_2 > 0$, (7) and (8) give $\lambda'(T) \leq 0$, $c'(T) > 0$, which correspond qualitatively to the parameters for molybdenum [16] for T, $K \in [300, T_m)$.

We consider nonstationary melting due to a surface heat source [1, 4] in simplified form, where the liquid effect is neglected, which is justified at the start, when the liquid is thin. Exact estimates have been made [17] on the thermal interaction between the liquid and solid. The boundary conditions are

$$x = x_0(t): q = q_0, \quad u = u_0; \tag{9}$$

$$x = x_m(t): \quad q = \tilde{q} - L_m(\gamma x_m'' + x_m'), \quad u = u_m \equiv \text{const}, \tag{10}$$

in which $\tilde{q} = q_1 + \tilde{k}(t)$, $q_1 \equiv \text{const}$, $|\tilde{k}(t)| \leqslant \tilde{k}_*$, $0 \leqslant t < \infty$.

The solution to (5) and (6) describes the heat transfer between the melting boundary and the thermal wave $\omega = \omega_0$ propagating against a relaxing background:

$$u_0 + k_1 = f_1/(x + l_1), \ \gamma q_0 = f_1 + \tau[\omega_0 \zeta'(\omega_0) - \zeta(\omega_0)], \ l_1, \ \omega_0 - \text{const}. \tag{11}$$

Conditions (9) and (11) correspond to continuity in the temperature and heat flux at the front, which moves with speed $x'_0(t) = f_1/\gamma\omega_0\tau$, $0 \leq t < \infty$. These conditions are met because $\omega = \text{const}$ gives a family of continuous thermal waves. We derive $\zeta(\omega)$ from (10), the energy balance at the phase boundary, whose temperature is the melting point. If $u_m + k_1 > 0$, then

$$\zeta = \omega \int_{\omega_m^0}^{\omega} z(\omega)\, d\omega + C_3, \quad \omega_m^0 = u_m + k_1,$$

$$z = \omega^{-2}[R(\omega) + \varkappa_1 C_1 + \varkappa_2 C_2], \quad \varkappa_1 = \cos A, \quad \varkappa_2 = \sin A, \quad A = L_1 \ln \omega,$$

$$L_1 R = \varkappa_1 R_1 + \varkappa_2 R_2, \quad R_1 = F_1 B_2 - F_2 B_1 - D_1, \quad R_2 = -F_1 B_4 + F_2 B_3 + D_2,$$

$$B_1 L_1 = -[\cos A]^{\omega}_{\omega_m^0}, \quad B_2(1 + L_1^2) = [\omega(\sin A - L_1 \cos A)]^{\omega}_{\omega_m^0},$$

$$B_3 L_1 = [\sin A]^{\omega}_{\omega_m^0}, \quad B_4(1 + L_1^2) = [\omega(\cos A + L_1 \sin A)]^{\omega}_{\omega_m^0}, \tag{12}$$

$$L_m F_1 = -f_1, \quad L_m F_2 = C_3(u_m + k_1), \quad D_1 = \int_{\omega_m^0}^{\omega} Q_1 \varkappa_2 d\omega,$$

$$D_2 = \int_{\omega_m^0}^{\omega} Q_1 \varkappa_1 d\omega, \quad \gamma\tilde{q}(\tau) = L_m Q_1(\omega).$$

The symbol $[f(y)]^{y}_{y^0}$ denoted $f(y) - f(y^0)$. When the surface source has constant output or is specified in terms of elementary functions (whose form can be envisaged from the expressions for z, D_1, and D_2), the (12) quadrature can be represented in finite form.

If $u_m + k_1 < 0$, we have a type (12) relation for $\zeta(\omega)$, in which $\varkappa_1$, and $\varkappa_2$ are polynomials in ω. This form is examined similarly and is omitted here.

We determine the equation of motion and the melting boundary speed from (5) and (12), which give

$$x_m = [f_1/(u_m + k_1)] + \zeta'(\omega_m), \tag{13}$$

$$N_m = N_1 + N_2\tau + \tilde{N}, \tag{14}$$

$$\gamma N_1 = f_1/(u_m + k_1 + L_m), \quad \gamma(u_m + k_1) N_2 = L_1(h_1 \cos bt - h_2 \sin bt),$$

$$b\gamma = L_1, \quad h_1 = C_2 s_1^0 - C_1 s_2^0, \quad h_3 = C_2 s_2^0 + C_1 s_1^0, \quad h_2 = h_3 - C_3,$$

$$\tilde{N} = M_1 + \tilde{M}, \quad M_1 L_3 = q_1[1 + L_1\tau(\sin bt - L_1^{-1}\cos bt)],$$

$$\gamma L_m \tilde{M} = \tau\left[\varkappa_{1m}\int_0^t \tilde{k}\varkappa_{1m}\tau^{-1}dt + \varkappa_{2m}\int_0^t \tilde{k}\varkappa_{2m}\tau^{-1}dt\right]. \tag{15}$$

The first term in (14) is constant, while the second is of relaxation type, and the third is uniquely related to the surface source output.

We assume that the boundaries migrate from left to right towards positive x and take $f_1 > 0$. At $t = 0$, the temperature pattern lies in the segment $x \in [x_m{}^0, x_0{}^0]$, whose ends are defined by formulas following from (5) and (13):

$$x_m^0 = (f_1 + h_2 + C_3)/(u_m + k_1), \quad x_0^0 + l_1 = f_1/\omega_0, \ \omega_0 = u_0^0 + k_1,$$
$$l_1 = -\zeta'(\omega_0), \quad u_0^0 = u_0(x_0^0), \quad \omega_0 = \delta_1(\omega_m^0 + L_m),$$
$$0 < \delta_1 < \omega_m^0/(\omega_m^0 + L_m) < 1, \quad \delta_2, \ \delta_3 \in (0, 1).$$

The relation $x_m{}^0 < x_0{}^0$ is ensured by choosing C_3. To meet the physically obvious condition $0 \leqq N_m < x'_0(t)$, it is necessary to meet the following bounds in accordance with $L_1 = [(u_m + k_1)/L_m]^{1/2}$:

$$\text{a)}\ L_1 > 1, \quad \frac{L_2}{L_1} < h_1 < h_2 < L_2, \quad N_1 > \max(\Delta_1, \Delta_2),$$
$$\Delta_1(1-\delta_1) = \delta_1\left(\frac{q_1 L_4}{L_3} + k_m\right), \quad \Delta_2 = \frac{q_1(L_1 - 2)}{L_3} - \Delta_3 + k_m; \tag{16}$$

$$\text{b)}\ L_1 < 1, \quad L_2 < h_2 < h_1 < \frac{L_2}{L_1}, \quad N_1 > \max(\Delta_1, \Delta_2),$$
$$\Delta_1(1-\delta_1) = \delta_1\left[\frac{q_1(2 - L_4)}{L_3} + k_m\right], \quad \Delta_2 = \Delta_3 - \frac{q_1 L_1}{L_3} + k_m. \tag{17}$$

In both cases, $(h_2 - h_1)L_1 = \delta_2 L_2(L_1 - 1)$, $k_m = 2\bar{k}_*/L_m$. Each system in (16) and (17) is nonconflicting and gives constraints on C_1, C_2, δ_1, $\bar{k}_*$.

Equation (14) describes the effects from the nonstationary surface source on the melting boundary speed; $\bar{k}(t)$ may be a nonmonotone bounded function, and then (15) shows that $\tilde{M}(t)$ has two components: a monotone (relaxation) one and a nonmonotone one, where $|\tilde{M}(t)| \leqq k_m$.

The initial pattern $x = x^0(u)$, $q = q^0(u)$, $x \in [x_m{}^0, x_0{}^0]$ is represented by (5) and (6) with $\tau = 1$ and is dependent on C_1, C_2, C_3.

From (1), a time-local partial solution has been obtained [12] for nonstationary melting. One can assume formally that this applies for any finite interval for which $a < \infty$, $x_0'(t) < \infty$, and in practice it is best to use it for the interval $t \in [0, n\gamma]$ representing a multiple of several relaxation periods, $n \leqq 5$. Let the (11) ahead of the wave at $t = 0$ occupy a finite interval $[x_0{}^0, x_1]$, where we take the right-hand end as $u_1 + k_1 = \delta_3(u_0{}^0 + k_1)$, and then in time $t_1 = \gamma n$, $n = -\ln \delta_3$ the wave travles $x_1 - x_0{}^0 = f_1(\delta_3{}^{-1} - 1)/(u_0{}^0 + k_1)$, which is determined by f_1 and δ_3. The solution is thus suitable up to t_1, at which the wave reaches $x = x_1$.

3. Evaporation. We apply a hodograph transformation [7, 15, 18] to the heat-transport equations in (2) form, i.e., we interchange the dependent and independent variables:

$$\tau_u = \Omega x_v, \quad \tau^2 x_u = \Omega a \gamma \tau_v,$$
$$\tau = \tau(u, v), \quad x = x(u, v), \quad a = a(u), \quad x_u\tau_v - x_v\tau_u \neq 0. \tag{18}$$

Usually, this transformation is employed to linearize an initial system composed to two quasilinear equations homogeneous in the derivatives and has been used effectively in gas dynamics [18] and transport theory [7] (reversal method). Here (18) remains quasilinear. An advantage of (18) by comparison with (1) is that the aspect is eliminated on the nonlinearity due to the thermophysical parameters being dependent on temperature.

The evaporation is caused by a surface heat source $\tilde{q} \equiv \text{const}$:

$$x = x_m: \frac{dx_m}{dt} = S_m(T_m), \quad q_m = q_s + L_m\left(\gamma\frac{d^2x_m}{dt^2} + \frac{dx_m}{dt}\right), \tag{19}$$

$$x = x_e: \frac{dx_e}{dt} = S_e(T_e), \quad q_e = Z_e\tilde{q} - L_e\left(\gamma\frac{d^2x_e}{dt^2} + \frac{dx_e}{dt}\right). \tag{20}$$

We consider the process between the melting and evaportion boundaries, where we incorporate the absorptivity and use a kinetic condition [2] relating the speed of each boundary to the corresponding transition temperature. We take a stationary temperature distribution and a relaxing heat flux in the melting zone:

$$\Omega\lambda(T_s)\frac{dT_s(x)}{dx} = -q_0, \quad q_s = q_0 + q_1\tau, \quad q_1 \neq 0.$$

This assumption is analogous to the simplification in a Stefan treatment [6] in which the temperature in the phase transition is taken as known and constant.

We introduce $\beta = u^{-1}$ instead of u and assume that

$$a = b_* u + \tilde{a}(\beta), \quad Z = Z_1 u; \quad b_*, \ Z_1 - \text{const}, \quad T \in [T_1, T_2],$$
$$\tilde{a}(\beta) = b_\varepsilon \beta^\varepsilon, \ \varepsilon \geqslant 0, \ b_* Z_1 \neq 0, \quad 0 < \beta_2 \leqslant \beta \leqslant \beta_1 < 1. \tag{21}$$

The summation is with respect to the repeated superscript ε. For $c \equiv$ const, (21) covers a linear temperature dependence for the absorptivity and thermal conductivity. We take $S(T) = \sigma(\beta)$ as analytic and such that $0 < \sigma(0) < \infty$. If in particular $S = \sigma_0 \exp(-E/T)$ [2], than $\sigma_0 = \sigma_e(0) = \sigma_m(0)$.

We construct the solution to (18) as a functional series:

$$x = x_* \ln\beta + x_\varepsilon(\xi)\beta^\varepsilon, \quad \tau/\Omega = \tau_*\beta^{-1} + \tau_\varepsilon(\xi)\beta^\varepsilon, \ \varepsilon \geqslant 0,$$
$$\xi = [v - w(\beta)] f(\beta), \ x_*, \ \tau_* - \text{const}, \ \xi \in [0, \xi_e], \ \beta \in (0, 1), \tag{22}$$
$$x_0 = (\tau_*\xi/f_0) + l_0, \ \tau_0 = h_0 - z_0\xi, \ x_*\tau_*^2 = B_* z_0.$$

The recurrent relations for $n \geqq 1$ are

$$x_n = B\xi f_n + l_n + H_{n-1}, \ Bf_0^2 = -\tau_*, \ B_* = \gamma b_* f_0, \ f_0 \neq 0, \tag{23}$$

$$\tau_n = A_n f_n + BC\int_0^\xi \varphi_{n-1} d\xi - \frac{nC}{f_0}\left(l_n\xi + \int_0^\xi H_{n-1} d\xi\right) + h_n + E_{n-1},$$
$$B_* C = \tau_*^2 f_0, \ H_{n-1}(0) = E_{n-1}(0) = 0, \ H_0 = 0, \tag{24}$$
$$A_n f_0^2 = C\xi[x_* + (n\tau_*\xi/2f_0)], \ \varphi = (\xi f'/f) - w'f, \ fg = 1,$$

$$H_{n-1} = F_{n-1} + A_{n-1}\varphi_{n-2} - \frac{(n-1)}{f_0}\left(\xi h_{n-1} + \int_0^\xi G_{n-1} d\xi\right),$$
$$-f_0\tilde{F}_{n-1} = \sum_{i=1}^{n-1} f_i x'_{n-i} + \sum_{i=1}^{n-2} \tau'_i \varphi_{n-2-i}, \quad G_{n-1} = \int_0^\xi \tilde{G}_{n-1} d\xi,$$
$$\tilde{G}_{n-1} = \tilde{E}_{n-2} - [\tau_*^2(n-1)x_{n-1}/B_*] + (z_0 f_{n-1}/f_0), \tag{25}$$
$$-\tilde{E}_{n-1} = B_*^{-1}\left(\tau_*^2 \sum_{i=1}^{n-1} \varphi_{i-1} x'_{n-i} + \sum_{i=1}^{n-2} \vartheta_i \psi_{n-i-1} + 2\tau_0\tau_*\psi_{n-1} + x_*\vartheta_{n-1}\right) +$$
$$+ (b_* f_0)^{-1}\left(b_* \sum_{i=1}^{n-1} f_i \tau'_{n-i} + \sum_{i=0}^{n-1} b_i g_{n-i-1}\right),$$
$$\vartheta_0 = 0, \ \vartheta = \tau^2, \ \psi_0 = x_*, \ \psi_n = nx_n + \sum_{i=1}^{n} \varphi_{i-1} x'_{n-i}.$$

The coefficients in the power-series expansion in β for f, φ, σ, and so on are denoted by the same symbol with appropriate subscripts such as $f = f_\varepsilon\beta^\varepsilon$, $\varepsilon \geqq 0$. The $E_{n-1}(\xi)$, $F_{n-1}(\xi)$ are calculated from (25)-type formulas.

This solution contains four arbitrary functions ℓ, h, f, and w together with the argument β, which enables one to satisfy (19) and (20). We take $\xi_m = 0$ to get for the first few coefficients that

$$x_* = \gamma\sigma_0, \ w_0 = \gamma q_1, \ \tau_* w_1 = \gamma(d_0 + \sigma_0 d_m), \ s\tau_* = -x_*,$$
$$\tau_*(w_0 + g_0\xi_e) = \gamma d_1, \ Bf_1 = \gamma(\sigma_{1e} - \sigma_{1m}) - z_0 s, \tag{26}$$
$$(w_0 + g_0\xi_e)(z_0 - h_0) = \gamma\sigma_0 d_e + \tau_*(w_1 + g_1\xi_e), \ l_1 = sh_0 + \gamma\sigma_{1m},$$

$$q_0 = \Omega d_0,\ Z_1 c_b T_b q = \Omega d_1,\ d = L x_*^2 / t_b \lambda_b T_b.$$

The general form for the recurrent formulas is

$$w_n = K_{n-1},\ l_n = sh_{n-1} + \frac{\gamma}{n}\sigma_{n,m} + M_{n-1},\ M_0 \equiv 0,\ n \geqslant 1,$$

$$(w_0 + g_0\xi_e)h_{n-1} - (\tau_*\xi_e/f_0^2)f_n = R_{n-1},\ \tau_*(n + f_0 w_0 + \xi_e - 1) \neq 0,$$

$$B\xi_e f_n - \frac{(n-1)}{bf_0}h_{n-1} = \frac{\gamma}{n}(\sigma_{n,e} - \sigma_{n,m}) + P_{n-1}.$$

The expressions for K_{n-1}, M_{n-1}, P_{n-1}, R_{n-1} are composed of coefficients corresponding to approximations preceding approximation n; these formulas are not given here. The result $\xi_e \in (0, 1)$ is defined by $0 < q_s^\infty < \infty$, and parameter Ω is uniquely related to the arbitrary constant τ_*: when we satisfy the initial conditions at the evaporation boundary $\tau_e^0 = 1$, $x_e^0 = 0$, we get $1 = \Omega u_e^0[\tau_* + \tau_\varepsilon(\xi_e)(\beta^0)^{\varepsilon+1}]$, $\beta^0 = 1/u_e^0$. The solution to (22) gives x_m^0, u_m^0 and the initial temperature pattern between the phase boundaries.

One can use the Weierstrass–Kovalevskaya majorant method to show that if $\tilde{a}(\beta)$, $\sigma(\beta)$, $\beta \in (0, 1)$ are analytic functions, the series (22)-(24) converge for $\xi \in (0, 1)$, $\beta \in (0, 1)$. The solution structure indicates that it is of boundary-layer type nd describes a nonstationary transition due to thermal relaxation from the initial temperature pattern between the phase boundaries to the limiting state $t \to \infty$.

We see from (22) and (26) that in the simplest approximation

$$x \simeq x_* \ln\beta + x_0 + x_1\beta,\ \tau/\Omega \simeq \tau_*\beta^{-1} + \tau_0,$$
$$w \simeq w_0 + w_1\beta,\quad f \simeq f_0 + f_1\beta \tag{27}$$

one already has information on how the thermal diffusivity varies with u (parameter b_*), the nonlinear absorvity (Z_1), the kinetic relations at the phase boundaries (σ_0, E_m, E_e), and the thermal conditions in the melting material (q_0, q_1). The major qualitative regularities in (27) are not altered by incorporating subsequent expansion terms. We subsequently put $c \equiv$ const.

At the phase boundaries, which move slowly, the temperature decreases over time in a relaxation fashion:

$$u(T_m) \simeq (\tau - \Omega h_0)/\Omega\tau_*,\ u(T_e) \simeq [\tau - \Omega(h_0 - z_0\xi_e)]/\Omega\tau_*,$$
$$\xi_e \simeq d_*(\gamma\sigma_0^2 - u_e^\infty b_*),\ \gamma q_e^\infty = \Omega\xi_e u_e^\infty,\ d_* c_* = d_1 - \tau_* q_1,\ \tau_* > 0. \tag{28}$$

The natural requirements $u_e > u_m > 1$, $\Omega > 0$ lead to the bounds

$$b_*(d_1 - \tau_* q_1) > 0,\quad D_0 \equiv d_1 - d_0 - \sigma_0(d_m + d_e) + c\sigma_0(E_e - E_m) > 0,$$
$$D_0 + (d_1 - \tau_* q_1)(d_1 - \sigma_0)\gamma\sigma_0 b_*^{-1} < 0. \tag{29}$$

To (29) we must add one of the conditions

$$\text{a)}\ b_* < 0,\quad \xi_e < \xi_*,\quad d_1 = \xi_*(\sigma_0 + d_1);$$
$$\text{b)}\ 0 < b_* < \gamma\sigma_0^2,\quad d_*(\gamma\sigma_0^2 - b_*) < \xi_*. \tag{30}$$

Compatibility between (29) and (30) is provided by suitable choice of d_0, d_1, q_1. The condition $\beta < 1$ will be met if the temperature scale is taken as the value at the melting boundary for $t \to \infty$.

The formula $N_e = \sigma_0 \exp(-E_e c/u_e)$ for the evaporation boundary speed shows that the asymptotic value ($t \to \infty$) is very much dependent on b_* and q_1. For $0 < b_* < \gamma\sigma_0^2$, if $d_1 - H < q_1\tau_* < d_1$, then $M_* \equiv dN_e^\infty/db^* > 0$, while if $q_1\tau_* < d_1 - H$, then $M_* < 0$; $H\gamma\sigma_0^2 = c_*$. For $b_* < 0$, if $d_1 < q_1\tau_* < d_1 - H$, then $M_* > 0$, while if $q_1\tau_* > d_1 - H$, then $M_* < 0$. This means that the N_e^∞ set up during the thermal relaxation and the N_m^∞, whose behavior is analogous, are dependent on da/du and on dq_s/dt, the rate of change in the heat flux in the melting zone.

These results show clearly that thermophysical-parameter nonlinearity has a marked effect on the nonstationary heat-transfer parameters in the phase transitions.

NOTATION

Dimensionless quantities: x, Cartesian coordinate; t, time; T, temperature; λ, thermal conductivity; c, bulk specific heat; q, specific heat flux; γ, heat-flux relaxation time; L, heat of phase transition in unit volume; $\tilde{q}$, set heat flux density at surface; N, phase boundary speed; Z, absorptivity; u and v, variables in hodograph plane; $s_1 = \cos A_m$, $s_2 = \sin A_m$, $A_m = A_m^0 + bt$, $L_1 = [(u_m + k_1)/L_m]^{1/2}$, $L_2L_3 = \gamma q_1 (u_m + k_1)$, $L_3 = L_m (1 + L_1^2)$, $L_4 = L_1 - \delta_2 (L_1 - 1)$, $\Delta_3 \gamma \omega_m^0 = \delta_2 L_2 (L_1 - 1)$, δ_2 and δ_3, arbitrary numbers in the range (0, 1); $c_* = cb_* \sigma_0 (E_e - E_m)$. Subscripts and superscripts: overbars for dimensional quantities; m and e, melting and evaporation phase boundaries; b, scale for dimensionless quantities; ε, summation; 0, initial value; ∞, asymptotic (t → ∞) value; prime ordinary differentiation; independent variable as subscript, partial differentiation.

LITERATURE CITED

1. N. N. Ryaklin and A. A. Uglov, Effects of Concentrated Energy Fluxes on Materials [in Russian], Moscow (1985), pp. 5-19.
2. S. I. Anisimov, Ya. A. Imas, G. S. Romanov, and Yu. V. Khodyko, Effects of High-Power Radiation on Metals [in Russian], Moscow (1970).
3. B. Ya. Lyubov and É. N. Sobol', Fiz. Khim. Obrab. Mater., No. 1, 12-26 (1979).
4. B. Ya. Lyubov and É. N. Sobol', Inzh.-Fiz. Zh., 45, No. 4, 670-686 (1983).
5. S. S. Filippov, B. N. Chetverushkin, and E. V. Shil'nikov, Dokl. An SSSR, 272, No. 4, 835-838 (1983).
6. A. V. Lykov, Thermal Conduction Theory [in Russian], Moscow (1967).
7. P. M. Kolesnikov, Transport-Theory Methods for Nonlinear Media [in Russian], Minsk (1981).
8. A. G. Goloveiko and A. D. Khudokormov, Izv. Vyssh. Uchebn. Zaved., Énerg., No. 3, 87-91 (1975).
9. I. A. Solov'ev, Inzh.-Fiz. Zh., 40, No. 2, 373-374 (1981).
10. O. N. Shablovskii, Inzh.-Fiz. Zh., 49, No. 3, 499-500 (1985).
11. O. N. Shablovskii, Inzh.-Fiz. Zh., 50, No. 6, 1037 (1986).
12. O. N. Shablovskii, Inzh.-Fiz. Zh., 52, No. 2, 309-316 (1987).
13. A. Visintin, IMA J. Appl. Math., 34, No. 3, 225-245 (1985).
14. Yu. S. Zav'yalov, Dokl. AN SSSR, 103, No. 5, 781-782 (1955).
15. L. S. Rozhdestvenskii and N. N. Yanenko, Quasilinear-Equation System and Their Applications in Gas Dynamics [in Russian], Moscow (1968).
16. N. N. Stolovich and N. S. Minitskaya, Temperature Dependence of the Thermophysical Parameters for Certain Metals [in Russian], Minsk (1975).
17. A. A. Uglov, I. Yu. Smurov, and A. G. Gus'kov, Fiz. Khim. Obrab. Mater., No. 3, 3-8 (1985).
18. R. Mises, The Mathematical Theory of Compressible Liquid Flows [Russian translation], Moscow (1961).